Research on Shape Adjustment of Active Reflector Based on Optimization Model

Yuexin Yang[1, *], Zhuoxun Chen[2], Xinyu Zhang[3], Shen Junyu[4], Li Mingyuan[3]

[1]College of Overseas Education, Changzhou University, Changzhou, Jangsu, 213164

[2]George School, Newtown, 18940

[3]School of Energy and Power Engineering, Northeast Electric Power University, Jilin, Jilin, 132012

[4]College of Computer, Northeast Electric Power University, Jilin, Jilin, 132012

*Corresponding author. Email: yangyuexin0318@163.com

Keywords: objective programming model, traversal method, optimization model, rotation coordinates.

Abstract: Aiming at the problem of determining the ideal paraboloid in FAST, this paper obtains the variation model of the working paraboloid based on the objective programming model, ergodic method, linear fitting, and spatial geometry knowledge. In this paper, the ideal paraboloid is as close as possible to the spherical reference surface, and the variation range of the distance between adjacent nodes and the radial expansion and contraction range of the actuator are used as constraints and target optimization model. Moreover, find the ideal paraboloid when the celestial body S to be observed is located at α = 36.795°, β = 78.169°. Meanwhile, a comparative analysis is made with the reference spherical receiving ratio.

1. Introduction

Since the Chinese astronomical community proposed the construction of the Sky Eye in 1994, the Chinese Academy of Sciences Astronomical Observatory and scientists from the astronomical community have completed the construction plan of the 500-meter-aperture spherical radio telescope, the China Sky Eye (FAST). Sky Eye, the radio telescope with the largest single-aperture and the highest sensitivity globally, has the country's independent intellectual property rights. Its appearance has promoted the progress of the country's scientific research and technology and contributed to the goal of the country's modernization and construction of a powerful country.

This paper builds a model to analyze the following problems. (1) The celestial body S is the object to be observed. When it is directly above the reference sphere, at this time, α = 0°, β = 90°. Synthesizing the adjustment factors of the reflective panel, the ideal paraboloid is given. (2) The celestial body S is the object to be observed. When it is located at α = 36.795°, β = 78.169°, the ideal paraboloid is given. In order to make the reflective surface as close to the ideal paraboloid as possible, the adjustment model of the reflective panel is established by adjusting and changing the telescopic amount of the actuator and at the same time limiting the variation range of the distance between adjacent main cable nodes. (3) Under the condition that the adjustment model of the second reflection panel has been established and solved, the receiving ratio of the feed source cabin after adjustment to the working parabolic surface is obtained, and it is compared with the receiving ratio of the reference reflecting spherical surface.

2. Establishment of a planning model with the smallest mean square error as a single objective

2.1 Establishment of the target planning model

When using the celestial eye to observe the target celestial body S, the parallel electromagnetic waves from the target celestial body are reflected and converged to the focal point P, where P is the center of the receiving plane feed cabin. At this time, the working paraboloid is considered an ideal paraboloid.

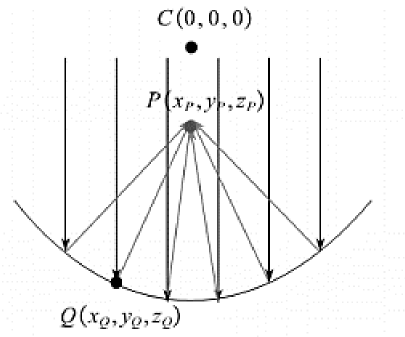

Figure 1 Schematic diagram of celestial body S signal emission

(1) Objective function

In order to obtain the ideal paraboloid, this paper assumes that the ideal paraboloid is infinitely close to the reference sphere in the working state. When the ideal paraboloid can be approximately regarded as a spherical shape, the mean square error of the main cable node in the working state and the main cable node in the reference state is used as the objective function to minimize the mean square error.

That is, the objective function is

$$arg_a \min \sqrt{\frac{\sum_{\vartheta} \| q_i - q_i{}^0 \|_2^2}{card\vartheta}} \tag{1}$$

$card\vartheta$ it represents the number of sets of main index node coordinates.

(2) Constraints

(1) Constraint 1: Constraints on the variation range of the main cable nodes

It is known from the title that after the main index node is adjusted, the distance between adjacent nodes may change slightly, and the change range does not exceed 0.07%. Set two adjacent main index nodes i and j. $(x_i{}^0, y_i{}^0, z_i{}^0)$ is the position of the main cable node in the reference state and is i the position of the main cable node in (x_i, y_i, z_i) the working state i.

$$d_0 = \sqrt{(x_i{}^0 - x_j{}^0)^2 + (y_i{}^0 - y_j{}^0)^2 + (z_i{}^0 - z_j{}^0)^2} \tag{2}$$

$$d = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \tag{3}$$

The distance between adjacent nodes can be $d - d_0$ represented. which is

$$-0.07\% d_0 \leqslant d - d_0 \leqslant 0.07\% d_0$$

(2) Constraint 2: Constraint of the radial expansion and contraction range of the actuator

In the reference state, the radial expansion and contraction of the top end of the actuator are 0. The radial expansion and contraction range of $-0.6 \sim +0.6\ m$ the actuator is.

$$q_i = h \frac{q_i{}^0}{\sqrt{x_i{}^0 + y_i{}^0 + z_i{}^0}} + q_i{}^0, \forall i \in Q, h \in [-0.6, 0.6] \tag{4}$$

$q_i{}^0$ is the position of the main cable node in the reference state and is i the position of the main cable node in q_i the working state i.

In summary, the goal planning model is obtained:

$$arg_a \min \sqrt{\frac{\sum_{\vartheta} \| q_i - q_i{}^0 \|_2^2}{card\vartheta}}$$

$$s.t\begin{cases} -0.07\%d_0 \leqslant d-d_0 \leqslant 0.07\%d_0 \\ q_i = h\dfrac{q_i{}^0}{\sqrt{x_i{}^0+y_i{}^0+z_i{}^0}} + q_i{}^0, \forall i \in Q, h \in [-0.6, 0.6] \end{cases} \tag{7}$$

2.2 Model solution

In the process of solving the above model, the iterative interval of the parabolic surface function cannot be accurately determined, so this paper specializes in the parabolic surface, starts with the standard paraboloid, sets up the parabolic surface equation, performs linear fitting on it, and deduces the iterative interval in reverse.

(1) The relationship between the parabolic surface and the spherical reference surface

It is known that the paraboloid equation in the standard state is $z = x^2 + y^2$. Therefore, the equation of the working paraboloid can be set to be $z = a(x^2 + y^2) + b$ the b approximate range by bringing the coordinates of the main cable nodes in the reference state into and fitting out. a

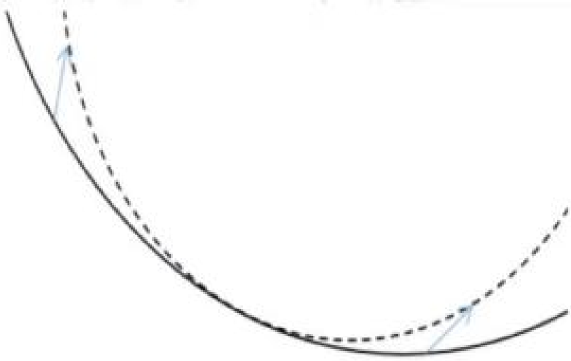

Figure 2 Schematic diagram of actuator extension and retraction

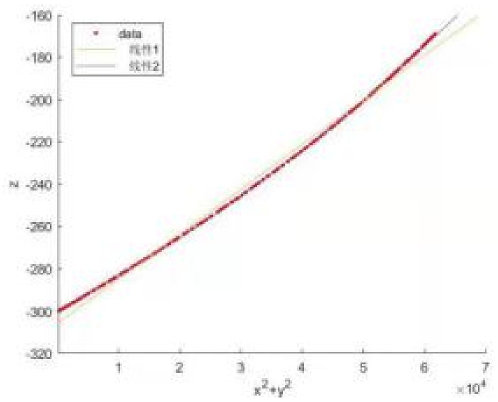

Figure 3 Schematic diagram of linear fitting

Since it cannot be ensured that each main cable node falls on the required paraboloid, the above model is optimized in this paper.

The main cable node corresponds to the point $q(x, y, z)$ on the working paraboloid and the reference sphere q_0. Due to the radial expansion and contraction of the bottom end of the actuator, it can be considered q_0 that q they are all on the straight line formed by the connection between the actuator and point C.

$$\begin{cases} x = x_0 \cdot \dfrac{z_0 \pm \sqrt{z_0{}^2 - 4ab(x_0{}^2 + y_0{}^2)}}{2a(x_0{}^2 + y_0{}^2)} \\ y = y_0 \cdot \dfrac{z_0 \pm \sqrt{z_0{}^2 - 4ab(x_0{}^2 + y_0{}^2)}}{2a(x_0{}^2 + y_0{}^2)} \\ z = z_0 \cdot \dfrac{z_0 \pm \sqrt{z_0{}^2 - 4ab(x_0{}^2 + y_0{}^2)}}{2a(x_0{}^2 + y_0{}^2)} \end{cases} \tag{5}$$

In this regard, this paper lists the following expressions.

$$\begin{cases} \dfrac{x}{x_0} = \dfrac{y}{y_0} = \dfrac{z}{z_0} \\ z = a(x^2 + y^2) + b \end{cases} \tag{6}$$

According to this expression, it will be (x, y, z) reduced to (x_0, y_0, z_0) the expression related to and, namely:

2.3 Model Results

An ideal paraboloid is obtained by fitting the data in Annex 1, and a three-dimensional diagram of the ideal paraboloid is made (as shown in Figure 7).

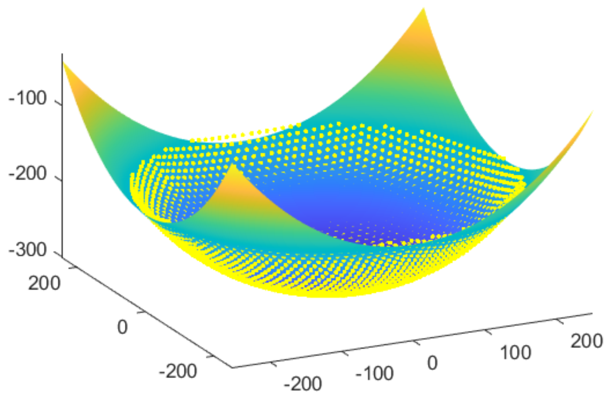

Figure 4 Ideal paraboloid

In order to enhance the accuracy of the data, all solutions a are b kept to four decimal places. Solve it with Matlab, and finally get the z axis coordinate of the vertex of the ideal paraboloid as - 300.9800 and the vertex as $(0, 0, -300.9800)$. Since the focus is on the straight line S C, and the straight line S C coincides with z the axis, the focal length of the paraboloid $300.9800-(1-0.466)R=140.7800$ is

$$z=0.0020(x^2+y^2)-300.9800 \tag{8}$$

3. Determination of ideal paraboloid after rotation

3.1 Establishment of the objective optimization model

The model used in the second question is the same as the first question, that is, the mean square error of the main cable node in the working state and the main cable node in the reference state is used as the objective function, and the range of node change and the radial expansion and contraction range of the actuator are used as constraints—planning model.

$$arg_a \min \sqrt{\frac{\sum_{\vartheta} \| q_i - q_i{}^0 \|_2^2}{card\vartheta}} \tag{9}$$

$$s.t\begin{cases} -0.07\%d_0 \leqslant d-d_0 \leqslant 0.07\%d_0 \\ q_i = h\dfrac{q_i{}^0}{\sqrt{x_i{}^0+y_i{}^0+z_i{}^0}} + q_i{}^0, \forall i \in Q, h \in [-0.6, 0.6] \end{cases} \tag{10}$$

3.2 Results of the model

Through the rotation of the matrix, the ideal paraboloid at α = 36.795°, β = 78.169° is obtained (as shown in the figure below).

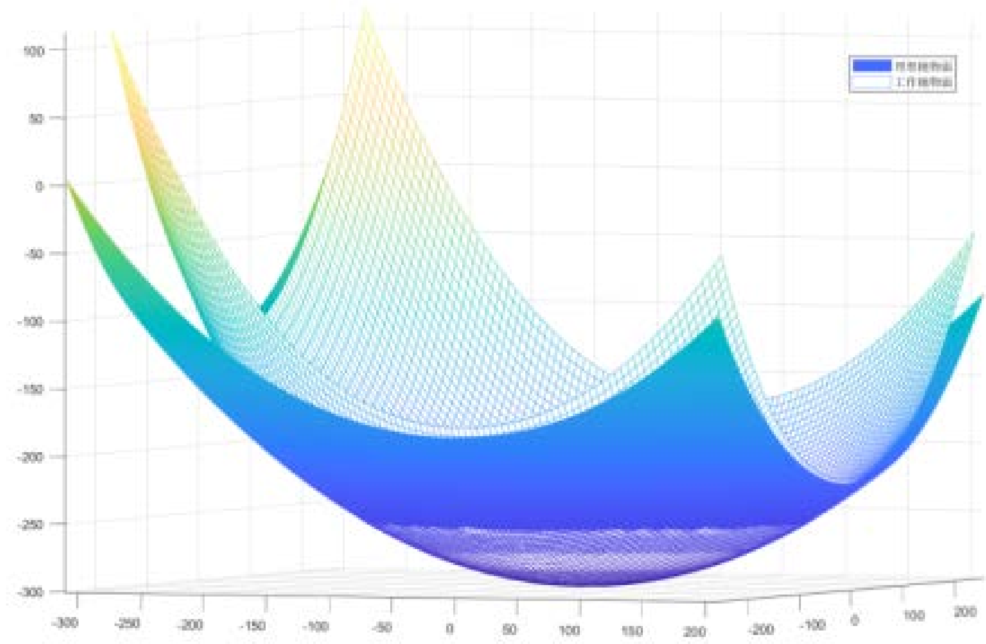

Figure 5 Comparison of ideal paraboloid and working paraboloid

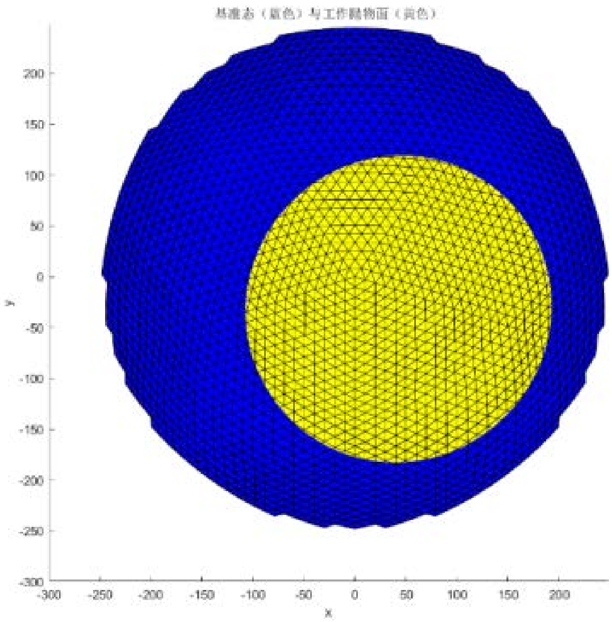

Figure 6 Schematic diagram of the reference sphere and the working paraboloid

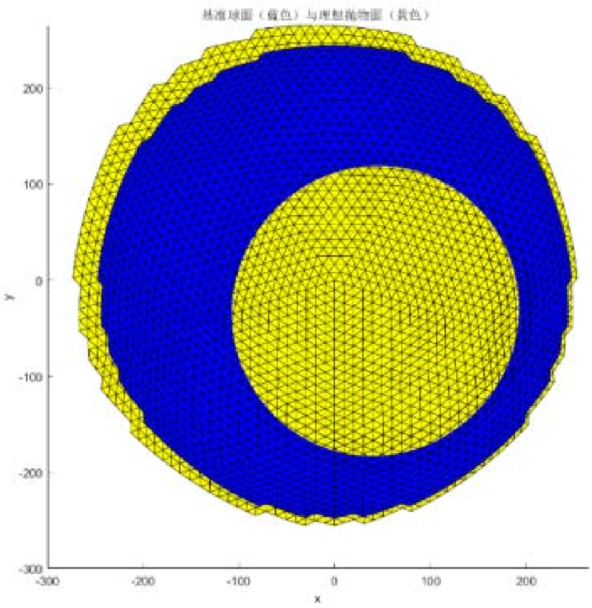

Figure 7 Schematic diagram of reference sphere and ideal paraboloid

The functional expression of the ideal paraboloid is

$$z = 0.001719x^2 + 0.001736y^2 + 0.03832x - 0.02876y - 300 \tag{11}$$

At the same time, the obtained vertex coordinates of the ideal paraboloid and the adjustment results of the main cable node number, position coordinates, and the expansion and contraction amount of each actuator within the 300-meter diameter of the rear reflection surface are saved.

Table 1The number of the main cable node, the position coordinates, and the results of the expansion and contraction of each actuator

Node number	X coordinate (m)	Y coordinate (m)	Z coordinate (m)	Telescopic amount (m)
A0	0.000	0.000	-300.017	0.384
B1	6.100	8.396	-299.827	0.393
C1	9.873	-3.208	-299.931	0.289
D1	0.000	-10.380	-299.892	0.329
…	…	…	…	…
E427	-187.569	157.016	-175.400	(0.600)
E428	-181.625	165.918	-173.450	(0.600)
E429	-175.612	174.225	-171.485	(0.600)
E430	-169.307	182.360	-169.357	(0.600)

4. Model evaluation and promotion

4.1 Advantages of the model

In solving the problem, this paper optimizes the algorithm of the model to make the obtained ideal paraboloid more accurate. The objective programming model established in this paper is simple and easy to understand and can use the model and the model solving algorithm. An ideal paraboloid is obtained, which shows the practicability of the model and the effectiveness of the algorithm.

4.2 Disadvantages of the model

Since it is difficult for a sphere to converge parallel beams to the same point in real life, the selection of the parabolic approximation in this paper is likely to cause certain errors. The traversal and iterative methods used in this paper take a long time to program, have a high degree of complexity and are not concise enough. Due to the lack of relevant data and the longitudinal movement of the actuator is not considered, the model built has certain limitations, which is easy to cause errors in the application of practical problems.

4.3 Generalization of the model

The most important thing in this model is the determination of the ideal paraboloid of the reflective panel and the adjustment of the active, reflective panel, which can be applied to the installation and adjustment of the photovoltaic power generation panel. For the tracking of the maximum power point of the photovoltaic array, the curved surface where the maximum power is located at each time point is calculated. And then adjust the direction of the power generation board to generate more electricity. In communication engineering, the lateral deflection and scattering of the antenna feed are solved based on the paraboloid. For example, the receiving antenna of a large billboard is inconvenient to rotate, and the antenna lobe is challenging to measure. The antenna surface profile can be divided into multiple paraboloids for adjustment.

References

[1] Si Shoukui, Sun Zhaoliang, Sun Xijing. Mathematical Modeling Algorithms and Applications [M]. National Defense Industry Press, 2015.

[2] Jiang Qiyuan, Xie Jinxing. Mathematical Modeling [M]. Beijing: Higher Education Press, 2